Published by Institute of Physics Publishing for SISSA/ISAS
Received: *February 06, 2002*
Revised: *April 10, 2002*
Accepted: *April 10, 2002*

D-branes, fluxes and chirality

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Abstract: We describe a topological effect on configurations of D-branes in the presence of NS-NS and RR field strength fluxes. The fluxes induce the appearance of chiral anomalies on lower-dimensional submanifolds of the D-brane worldvolume. This anomaly is not associated to a dynamical chiral fermion degree of freedom, but rather should be regarded as an explicit flux-induced anomalous term (Wess-Zumino term) in the action. The anomaly is cancelled by an inflow mechanism, which exploits the fact that fluxes can act as sources of RR fields. We discuss several applications of this flux-induced anomaly; among others, its role in understanding anomaly cancellation in compactifications with D-branes and fluxes, and the possibility of phase transitions where a chiral fermion disappears from the D-brane world-volume spectrum, being replaced by an explicit Wess-Zumino term. We comment on the relation among different mechanisms to obtain four-dimensional chirality in string theory.

Keywords: D-branes, Anomalies in Field and String Theories.

JHEP04(2002)016

Contents

1. Introduction

Compactifications of type-II string theory/M-theory with field strength fluxes turned on (see e.g. [1]–[11]) are an interesting class of constructions which may shed new light on several questions of phenomenological relevance. For instance, the observation that fluxes lead to warped internal metric suggests the models may be used to generate exponential hierarchies [5, 7, 9], following the proposal in [12], yielding a possible solution for the hierarchy of scales in particle physics. On the other hand, the observation that the presence of fluxes induces tree-level potential terms for diverse moduli [4, 6], including even the dilaton, provides a possible canonical mechanism to stabilize them [5, 9, 10].

Several properties of type-II compactifications with fluxes have been studied recently. In particular, the consistency conditions for the configurations, the amount of supersymmetry preserved, and also the effective action for the closed string sector fields (e.g. potential for moduli). However the physics of D-branes, namely of open string sectors, in compactifications with fluxes has been much less analyzed. There are several indications that such physics is nevertheless extremely interesting; in fact D-branes in the presence of fluxes present interesting phenomena, for instance the dielectric effect [13]. Their systematic analysis is however difficult given the incomplete knowledge of non-abelian D-brane actions, and the difficulties to obtain worldsheet results in situations with RR fluxes.

In this paper we center on a particular effect whose analysis is relatively simple, since it involves only topological couplings. Yet the consequences of the effect are extremely interesting. In section 2, we show that, in the presence of suitable NS-NS and RR field strength fluxes, D-branes develop a chiral anomaly localized at lower-dimensional submanifolds of their volume. This is derived by first showing there exists an anomaly inflow from the D-brane world-volume towards such lower-dimensional slices, which signals the existence of an anomaly source. The source of the anomaly is not, contrary to other familiar situations, a chiral fermion degree of freedom, but rather an explicit anomalous interaction developed by the D-brane in the presence of the fluxes.

In section 3 we present two applications of this effect. It plays a key role in understanding of anomaly cancellation on the world-volume of D-brane probes in certain configurations with fluxes, and of cancellation of anomalies in chiral compactifications with fluxes. In section 4 we discuss brane/flux transitions and use them to generate transitions on D-brane world-volumes, in which a dynamical chiral fermion disappears from the spectrum, leaving behind an explicit Wess-Zumino term. Section 5 discusses how to generate global gauge anomalies using fluxes. Section 6 concludes with a discussion of the interrelation among different mechanisms to generate four-dimensional chirality in string theory.

2. Chirality from fluxes

In this section we discuss the effect of flux-induced anomaly on D-branes in the presence of fluxes. We start with a review of the anomaly inflow mechanism in section 2.1. In section 2.2 we show that in configurations of D-branes and NS-NS and RR fluxes, there exists an inflow of anomaly from the D-brane worldvolume towards lower-dimensional slices of its volume. The inflow signals the existence of an anomaly source, which we identify as an explicit anomalous interaction, rather than a dynamical fermion degree of freedom.

2.1 The anomaly inflow mechanism

Let us give a simplified review of the anomaly inflow mechanism for D-branes, as discussed in [14]. Consider a D-brane, which couples to the RR fields through the action

$$S_{\mathrm{D}p} = \int_{\mathrm{D}p} \mathcal{C} \wedge Y(F,R)\,, \tag{2.1}$$

where $\mathcal{C}$ is a formal sum of RR forms of different degrees, and Y is a closed gauge invariant form depending on the worldvolume gauge field strength and curvature, in fact

$$Y = \mathrm{ch}\,(F)\,\hat{A}(R)^{1/2}\,, \tag{2.2}$$

where ch is the Chern character and $\hat{A}$ is the A-roof genus. In what follows, wedge products will be implicit.

We will be interested in situations where the RR fields have a source (different form the D-brane itself), so that the field strength $\mathcal{G}$ obeys

$$d\mathcal{G} = \mathcal{Z}\,, \tag{2.3}$$

where $\mathcal{Z}$ is a source form (in fact a formal sum of them), which is often a bump form, a delta-function localized on the core of the source (typically, but not necessarily, a D-brane).

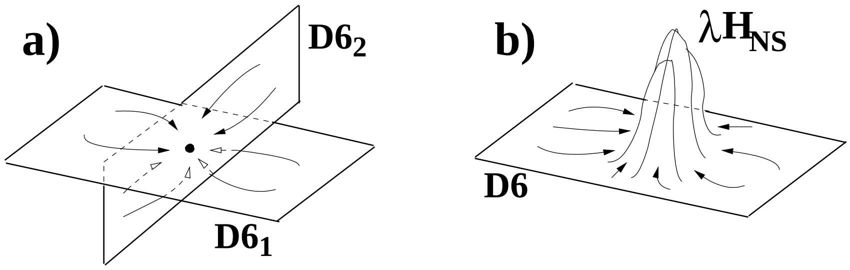

Figure 1: Anomaly inflow for a) D6-branes intersection over a four-dimensional submanifold of their world-volume; b) D6-brane in the presence of longitudinal $\lambda H_{\rm NS}$ flux.

In this situation, $\mathcal{C}$ is not well defined, so the D-brane coupling (2.1) is better defined by

$$S_{{\rm D}p} = \int_{{\rm D}p} \mathcal{G}\, Y^{(0)}(F,R)\,, \tag{2.4}$$

where we have used the Wess-Zumino descent relation notation, i.e. for a closed gauge invariant form Y, we define $Y = dY^{(0)}$, $\delta Y^{(0)} = dY^{(1)}$, where δ represents a gauge variation.

In this situation, the action (2.4) is not gauge invariant, its gauge variation is given by

$$\delta S_{{\rm D}p} = \int_{{\rm D}p} \mathcal{G}\, \delta Y^{(0)}(F,R) = \int_{{\rm D}p} d\mathcal{G}\, Y^{(1)}(F,R) = \int_{{\rm D}p} \mathcal{Z}\, Y^{(1)}(F,R)\,. \tag{2.5}$$

Recalling the familiar interpretation of $Y^{(1)}$ as the anomaly descending from an anomaly polynomial $Y(F,R)$, it is useful to interpret this last expression by saying that there exists an inflow of chiral anomaly from the Dp-brane wolume towards the core of the source (or rather towards its intersection with the Dp-brane volume), with density given by the source form $\mathcal{Z}$. That is, the Dp-brane action is no longer gauge invariant, and its gauge variation is localized on the core of $\mathcal{Z}$. If $\mathcal{Z}$ is not bumpy the inflow at each lower-dimensional slice is proportional to the magnitude of $\mathcal{Z}$.

This lack of gauge invariance must be compensated by a counteracting effect, arising at the core of the source Z. As we discuss below, the usual situation is that a dynamical chiral fermion arises, whose anomaly is cancelled by this inflow. In the following section we argue there exist situations where no such dynamical fermion arises, but instead there exists an explict Wess-Zumino interaction on the Dp-brane world-volume.

A familiar situation where the anomaly inflow is cancelled by the appearance of a dynamical chiral fermion degree of freedom is intersecting D-branes, see [15]. For concreteness consider two stacks of D6-branes intersecting over a four-dimensional subspace of their world-volumes, figure 1a. By a slight generalization of the above argument (see [14]), the world-volume action of each D6-brane stack is not gauge invariant due to the presence of the other D6-brane stack, which acts as a source. This results in an overall gauge variation

$$\delta S_{{\rm D6}_1+{\rm D6}_2} = \int [\,Y(F_1,R)\,Y(F_2,R)\,]^{(1)}\,\delta_I\,, \tag{2.6}$$

where δ_I is a bump 6-form localized at the four-dimensional intersection, and F_1, F_2 are the curvatures of the gauge bundles on the D6-brane stacks. Namely there is an inflow of chiral anomaly from the two sets of D6-branes towars their intersection.

On the other hand, the intersection of D6-branes leads to a chiral four-dimensional fermion, localized at the intersection. Such state can be seen to arise by direct quantization of the sector of open strings stretched between the D6-brane stacks, and therefore transforms in the bi-fundamental representation of the unitary gauge groups on the D6-branes. The triangle anomaly associated to that fermion is

$$[\,\mathrm{ch}\,(F_1)\,\mathrm{ch}\,(F_2)\,\hat{A}(R)\,]^{(1)} \tag{2.7}$$

and is localized at the intersection. Hence it precisely cancels the above anomaly inflow (2.6).

For future comparison, it is interesting to consider the inflow mechanism in compact models with intersecting branes. For instance, we consider type-IIA theory compactified on a Calabi-Yau threefold $\mathbf{X}_3$, with stacks of N_a D6-branes wrapped on 3-cycles $\mathbf{\Pi}_a$. Models of this kind with $\mathbf{X}_3 = \mathbb{T}^6$ (or orbifolds/orientifolds thereof) as the internal space have been considered e.g. in [16]–[20], and the following inflow picture was described in [18].

Localized anomalies due to chiral fermions at the intersections of D6-branes are cancelled by the above inflow mechanism. However, since the D6-brane worldvolume (transverse to the four-dimensional intersections) is compact, global consistency requires that the inflows at different intersections add up to zero (inflows to some intersections are compensated by outflows from other intersections). This constraint, which follows from cancellation of RR tadpoles, implies that the overal anomalies from fermions at intersections cancel in the ordinary four-dimensional sense. In particular, cubic non-abelian triangle anomalies automatically vanish, while U(1) — non abelian and U(1)-gravitational mixed anomalies cancel via a Green-Schwarz mechanism [18, 20].

2.2 Chirality from fluxes

In this section we turn to a novel situation where the inflow mechanism plays a key role, but contrary to the previous case does not imply the appearance of dynamical chiral fermion degrees of freedom.

2.2.1 The inflow

The basic observation stems form the fact that certain combinations of NS-NS and RR fluxes couple to RR fields in the same way as D-branes do. In particular, type-II theories contain couplings

$$\int_{\mathbf{X}_{10}} H_{\mathrm{NS}}\, G_{6-p}\, C_{p+1} \tag{2.8}$$

with G_{6-p} the field strength of the RR field C_{5-p}, and p even/odd for IIB/IIA theories. Hence, configurations with net flux for $H_{\mathrm{NS}}G_{6-p}$ are sources for the RR field C_{p+1}. The coupling is normalized such that one unit of flux carries one unit of Dp-brane charge. A familiar example of such coupling is the type-IIB interaction $\int H_{\mathrm{NS}}H_{\mathrm{RR}}C_4^+$, which endows $H_{\mathrm{NS}}H_{\mathrm{RR}}$ with D3-brane charge.

Following the general argument above, combinations of fluxes acting as sources for RR fields may induce anomaly inflows on D-branes present in the background. For concreteness, let us consider a (massive) type-IIA configuration with non-trivial value for the 0-form field strength λ, i.e. cosmological constant, and $H_{\rm NS}$ flux. Due to the coupling

$$\int_{\mathbf{X}_{10}} \lambda\, H_{\rm NS}\, C_7 \tag{2.9}$$

the flux acts as a source for the RR field C_7, namely

$$dG_2 = \lambda\, H_{\rm NS}\,, \tag{2.10}$$

where G_2 is the fields stregnth of C_1, the IIA RR 1-form.

Consider a D6-brane in this background. Among its Chern-Simons couplings, written as in (2.4), its world-volume action contains

$$S_{\rm D6} = \int_{\rm D6} G_2\, [\, {\rm ch}\,(F)\, \hat{A}(R)^{1/2}\,]^{(0)}\,. \tag{2.11}$$

Which is not gauge invariant, its gauge variation being given by

$$\delta\, S_{\rm D6} = \int_{\rm D6} dG_2\, [\, {\rm ch}\,(F)\, \hat{A}(R)^{1/2}\,]^{(1)} = \int_{\rm D6} \lambda\, H_{\rm NS}\, [\, {\rm ch}\,(F)\, \hat{A}(R)^{1/2}\,]^{(1)}\,. \tag{2.12}$$

Namely, in the presence of suitable flux, there exists an anomaly inflow from the D6-brane volume towards the four-dimensional core of the $H_{\rm NS}$ flux background, see figure 1b. In general $H_{\rm NS}$ may not be bumpy, so we interpret the density of inflow anomaly at a given four-dimensional slice to be the magnitude of $\lambda H_{\rm NS}$ at such slice.

It is straightforward to cook up other configurations leading to inflows towards two-, four-, six-dimensional subspaces. Given the structure of (2.8) and the world-volume Chern-Simons coupling

$$\int_{{\rm D}q} G_{8-p}\, Y^{(0)}(F,R) \tag{2.13}$$

a Dq-brane in the presence of longitudinal $H_{\rm NS}G_{6-p}$ flux develops anomaly inflow towards $(p+q-8)$-dimensional subpaces. Hence for instance, a D7-brane in the presence of longitudinal $H_{\rm NS}$, $H_{\rm RR}$ 3-form fluxes develops inflow towards two-dimensional subspaces; a D9-brane in the presence of $H_{\rm NS}$ and 1-form field strength fluxes develops inflows towards six-dimensional subspaces.

2.2.2 Absence of dynamical chiral fermions

The question we would like to address is how this anomaly is cancelled, namely what effect takes place at the core of the flux background. Notice that the anomaly inflow is "almost" that necessary to cancel the anomaly of a fermion localized at the core of the $H_{\rm NS}$ flux. This is true for the pure gauge anomalies, since the inflow of anomaly equals $[{\rm ch}\,(F)]^{(1)}$, a fermion anomaly. However, the pure/mixed gravitational anomaly is not quite right to be cancelled by a dynamical fermion degree of freedom, being $\hat{A}^{1/2}$ instead of $\hat{A}$. This is a first hint suggesting the anomaly will not be cancelled by a dynamical chiral fermion.[1]

[1]Recall that in the case of intersecting D-branes the gravitational anomaly is fully reproduced thanks to the inflow from both D-branes towards the intersection. In the present case, one of the contributions is lacking due to the absence of Chern-Simons couplings in the "volume" of the flux.

A second piece of evidence in this direction is that a dynamical fermion with anomaly cancelling against the inflow should be transforming in the fundamental representation of the D-brane gauge group, and be a singlet under any other gauge factor. This contrasts with the familiar situation where degrees of freedom associated to D-branes transform in *bi*-fundamental or adjoint representations. This observation excludes the possibility of obtaining the desired chirality from a fermion arising from a non-zero index of the Dirac operator of D-brane worldvolume fermions, since these transform in adjoint representations, and so would whatever fermion zero modes they give rise to.

In fact, since fundamental representations arise from open strings with one endpoint on the D-brane, the main problem is where is the other endpoint? Despite the lack of a solvable worldsheet CFT in the presence of RR fluxes, the worldsheet description is valid and reliable, at least for dilute fluxes. Hence an additional endpoint must exist, and it is highly unlikely that it somehow "ends" on, or dissipates into, the flux.

In fact this issue can be studied in more detail by considering a situation where the RR flux admits a geometric description in M-theory. To this purpose, consider one D6-brane along the directions 0123456, in the presence of one unit of longitudinal $H_{\rm NS}$ and F_2 fluxes, along 234 and 56, respectively. In the M-theory lift, the former lifts to a G_4 flux turned on along the directions 234 and the M-theory circle. To simplify the rest of the lift, consider the directions 56 to parametrize a 2-sphere, pierced by the non-zero F_2 flux. The 2-sphere lifts to a 3-sphere, with the twist in the Hopf fibration reproducing the F_2 flux. Moreover, to reproduce the D6-brane charge, the M-theory circle should degenerate at a point in 789. Hence, the geometry is not simply $\mathbf{M}_8 \times \mathbb{S}^3$, but rather a fibration of squashed $\mathbb{S}^3$'s over $\mathbb{R}^3$ (parametrized by 789), times M_5 (along 01234). The squashing is determined by the property that the circle fiber in $\mathbb{S}^3$ is fibered over $\mathbb{R}^3$ such that it is a (different) Hopf fibration over the angular $\mathbb{S}^2$ in $\mathbb{R}^3$. Overall the topology is that of a cone over a 5-manifold wich is a circle fibration over $\mathbb{S}^2 \times \mathbb{S}^2$ with Chern classes equal to one. At the tip of the cone only one of the $\mathbb{S}^2$'s goes to zero volume.

Notice that the geometry is completely smooth (in fact, comparing this topology with [21] we learn that the geometry is a resolved conifold). Since the lift of the configuration is completely smooth, there is no obvious room to generate a chiral fermion from, say, some wrapped M2-brane state, which is the required kind of object to be charged under the gauge field. This is our last piece of evidence for the non existence of dynamical fermions in such situations.

2.2.3 Origin of the anomaly

Our proposal to cancel the chiral anomaly inflow is that the flux induces an explicitly anomalous term in the D-brane worldvolume, of exactly the right form. The key observation is that the D-brane action contains topological couplings to the $B_{\rm NS}$ field arising from the Chern-Simons coupling

$$\int_{{\rm D}q} \mathcal{G}\,[\, e^{(F-B_{\rm NS})}\,\hat{A}(R)^{1/2}\,]^{(0)}\,. \qquad (2.14)$$

For concreteness let us consider the previous situation of a D6-brane in the presence of a $\lambda H_{\rm NS}$ background. The D6-brane contains a coupling

$$\int_{\rm D6} \lambda\, B_{\rm NS}\, [\,{\rm ch}\,(F)\,\hat{A}^{1/2}\,]^{(0)}\,. \tag{2.15}$$

Notice this term is different from the ones involved in the inflow. Moreover, it is explicitly non-gauge invariant, its gauge variation being

$$\int_{\rm D6} \lambda\, B_{\rm NS}\, \delta[\,{\rm ch}\,(F)\,\hat{A}^{1/2}\,]^{(0)} = \int_{\rm D6} \lambda\, H_{\rm NS}\, [\,{\rm ch}\,(F)\,\hat{A}^{1/2}\,]^{(1)} \tag{2.16}$$

which is exactly of the form required to cancel the anomaly generated by the inflow.

Notice that this mechanism avoids the problems mentioned above. In particular the interaction induced by the presence of the flux provides the gauge and gravitational anomalies required to cancel the inflow.

The bottomline is that in the kind of situation we have considered, the inflow is cancelled not by a dynamical degree of freedom, but rather by an explicitly anomalous interaction induced by the flux.

This effect is indeed surprising, in particular in compactified models (see section 3.2 below). In fact it allows the construction of say four-dimensional models with chiral fermions in an anomalous representation of the gauge group. The theory is however consistent due to the existence of an explicit Wess-Zumino interaction in the effective action, which cancels the residual anomaly. The same applies to pure gravitational anomalies in two- and six-dimensional compactifications exploiting this mechanism.

Two comments are in order: first, the string construction ensures that this Wess-Zumino interaction is of microscopic origin, and is not an effective description of a dynamical degree of freedom (i.e. a situation along the lines of [22]). This situation is indeed surprising and to our knowledge not previously discussed in string theory. Second, the triangle anomalies cancelled by this mechanism are both reducible and irreducible, hence the mechanism is in principle not related to a Green-Schwarz anomaly cancellation mechanism (which only applies to reducible anomalies).

3. Applications

In this section we turn to two short applications of the effect we have discussed.

3.1 Probe analysis of tadpole vs anomaly cancellation

Introduction of an allowed brane probe in a consistent string background should result in a consistent world-volume field theory on the probe. In [23] it was proposed that tadpole cancellation conditions manifest as anomaly cancellation conditions on suitable brane probes. In this section we illustrate than on consistent backgrounds with fluxes cancellation of anomalies on brane probes may require the above flux-induced Wess-Zumino term in a crucial way.

Consider a prototypical example of compactification with fluxes, for instance F-theory compactified on a Calabi-Yau fourfold $\mathbf{X}_4$, elliptically fibered over $\mathbf{B}_3$. The C_4^+ tadpole $N_\chi = -\chi(\mathbf{X}_4)/24$ in [24] can be cancelled by a number $N_{\rm D3}$ of D3-branes and/or turning on NS-NS and RR 3-form fluxes with $N_H = \int_{B_3} H_{\rm NS} H_{\rm RR}$, with $N_\chi + N_{\rm D3} + N_H = 0$.

Following [23] one may test tadpole cancellation by introducing a D7-brane probe (or a stack of n such D7-probes) wrapped on the base $\mathbf{B}_3$, and testing the cancellation of two-dimensional gauge anomalies on the non-compact piece of the D7-probe worldvolume. The curvature of $\mathbf{B}_3$, and the background type-IIB (p,q) 7-branes induces a gauge anomaly of N_χ units on the two-dimensional non-compact piece of the D7-brane worldvolume (see below for an explicit example), in conventions where the anomaly of a left-handed Majorana-Weyl fermion in the fundamental of $\mathrm{U}(n)$ is $+1$. In the absence of fluxes, the intersections between $N_{\rm D3}$ D3-branes and the D7-probe lead to $N_{\rm D3}$ left-handed Majorana-Weyl fermions in the fundamental, so that the total anomaly cancels, since $N_\chi + N_{\rm D3} = 0$. In the presence of N_H units of flux, there are not enough fermions form intersections between the D3-branes and the D7-probe. Happily, the D7-probe effective action also contains an explicit Wess-Zumino term, induced by the presence of the flux, via the coupling

$$\int_{\rm D7} B_{\rm NS}\, H_{\rm RR}\, [\, {\rm ch}\,(F)\, \hat{A}(R)^{1/2}\,]^{(0)}\,. \qquad (3.1)$$

In the effective 2d probe theory this coupling gives rise, to a term $N_H \mathcal{L}_{\rm WZ}$, with $\mathcal{L}_{\rm WZ}$ the Wess-Zumino lagrangian.[2] This non-gauge invariant term cancels the remaining anomaly, and renders the probe worldvolume theory consistent.

An explicit example is given by taking $\mathbf{X}_4$ to be $\mathbf{K3}\times\mathbf{K3}$. Equivalently, type-IIB theory compactified on $\mathbf{K3}\times\mathbb{P}_1$ with 24 (p,q) 7-branes wrapped on $\mathbf{K3}$ and located at points in $\mathbb{P}_1$. Since $\chi(\mathbf{X_4}) = 24\times 24$, there exists a C_4^+ tadpole of $N_\chi = -24$ which must be cancelled by introducing D3-branes and/or fluxes with $N_{\rm D3} + N_H = 24$.

Let us recover this condition by analyzing cancellation of gauge anomalies on a D7-brane probe wrapped on $\mathbf{K3}\times\mathbb{P}^1$. The intersections of the D7 probe with the (p,q) 7-branes lead each to a chiral 6d fermion propagating on $\mathbf{K3}$. Due to the non-zero index of the Dirac operator on $\mathbf{K3}$, each such six-dimensional fermion gives a chiral right-handed MW two-dimensional fermion, so that we get a total two-dimensional anomaly of -24 on the D7-brane probe worldvolume. In the absence of flux, this anomaly can only be cancelled by dynamical fermions arising from intersections between D3-branes and the D7-brane probe, hence one requires $N_{\rm D3} = 24$. In the presence of N_H units of flux, we are left with $-24 + N_{\rm D3}$ units of 2d anomaly, but the flux-induced Wess-Zumino term contributes an additional amount of N_H units of two-dimensional anomaly, hence consistency of the probe world-volume theory requires $N_{\rm D3} + N_H = 24$, and reproduces the tadpole condition.

[2] By Wess-Zumino lagrangian here we refer to any non-gauge invariant interaction whose gauge variation is the appropriate anomalous term (see e.g. [25]. It is typically defined by picking a one dimension higher manifold $\mathbf{X}_{\rm D+1}$ with boundary the spacetime of interest, and has the typical form $\mathcal{S}_{\rm WZ} = \int_{\mathbf{X}_{\rm D+1}} Y(0)(F,R)$. By default we consider Wess-Zumino terms with gauge fields traced in the fundamental representation).

It is straightforward to consider other compactifications where fluxes contribute to tadpole cancellation, and verify the crucial role of the flux-induced WZ terms in the physics of D-brane probes. In fact, the present work originated from the study of the interplay between tadpole cancellation and anomaly cancellation on brane probes in compactifications with fluxes.

3.2 Application to model building

Type-II compactifications with RR and NS-NS fluxes are a promising class of models whose exploration is still in progress. In this section we illustrate that in models leading to chiral lower-dimension theories, the flux-induced WZ terms play a crucial role in understanding anomaly cancellation from the lower-dimensional viewpoint.

In order to illustrate this point, we consider a particular class of models. For concreteness we center on compactifications of type-IIA theory on a Calabi-Yau threefold $\mathbf{X}_3$ with N_a D6-branes wrapped on homology 3-cycles $[\Pi_a]$, and NS-NS flux $H_{\rm NS}$ turned on with total homology class $[H_{\rm NS}]$, and in the presence of a cosmological constant λ. Compactifications of this kind in the absence of fluxes have been considered in [16]–[20]. Notice that these compactifications are typically non supersymmetric, but supersymmetric configurations could be obtained by introducing O6-planes [20]. Study of supersymmetric models lies beyond the scope of the present paper, which centers on more topological aspects.

Extending the analysis in [18], we now derive the RR tadpole cancellation conditions. The action for the RR 7-form field is

$$S = \int_{M_4\times X_3} dC_7 \, * dC_7 + \sum_a N_a \int_{M_4\times \Pi_a} C_7 + \int_{M_4\times X_3} \lambda \, H_{\rm NS} \, C_7 \, . \qquad (3.2)$$

The equation of motion is

$$dH_2 = \sum_a N_a \, \delta(\Pi_a) + \lambda \, H_{\rm NS} \, , \qquad (3.3)$$

where H_2 is the field strength of the RR 1-form, and $\delta(\Pi_a)$ is a bump 3-form on $\mathbf{X}_3$ with support on Π_a. The equation in homology reads

$$\sum_a N_a \, [\, \Pi_a \,] + \lambda \, [\, H_{\rm NS} \,] = 0 \, . \qquad (3.4)$$

The low energy four-dimensional theory is generically chiral, with chiral fermions arising from D6-brane intersections. The gauge group is $\prod_a \mathrm{U}(N_a)$, and the chiral fermion content is given by

$$\sum_{a<b} I_{ab} (\Box_a , \overline{\Box}_b) \, , \qquad (3.5)$$

where $I_{ab} = [\Pi_a] \cdot [\Pi_b]$ is the intersection number, with its sign specifying the fermion chirality.

In the presence of fluxes, the above fermion content is explicitly anomalous. In particular, the $\mathrm{SU}(N_a)^3$ cubic anomalies are given by $\sum_b N_b I_{ab}$ which need not vanish. Happily we are now familiar with the presence of explicit flux-induced four-dimensional Wess-Zumino terms, in this case arising from the coupling $\int_{\rm D6} B_{\rm NS} Y^{(0)}$. In the present case the four-dimensional anomalous interaction has the form

$$S_{\rm WZ} = \sum_a I_{aH} \int_{M_4} \mathcal{L}_{{\rm WZ},a} \,, \tag{3.6}$$

where $I_{aH} = \lambda\,[\Pi_a]\cdot[H_{\rm NS}]$ and $\mathcal{L}_{{\rm WZ},a}$ is the Wess-Zumino lagrangian corresponding to a fermion in the fundamental of $\mathrm{U}(N_a)$ (including the $\mathrm{U}(1)$ factor), and singlet under the remaining factors.

The total $\mathrm{SU}(N_a)^3$ cubic anomaly is therefore $\sum_b N_b I_{ab} + I_{aH}$, which cancels by using the tadpole condition (3.4). It is easy to check that mixed $\mathrm{U}(1)$ — non-abelian anomalies cancel by a Green-Schwarz mechanism, exactly as in [18], while mixed gravitational anomalies vanish automatically.

It is easy to find applications of the use of fluxes in compact model building, to avoid the appearance of unwanted gauge groups and matter contents associated to extra D-brane stacks, typically present in all realistic D-brane compactifications. For instance, in the realistic models obtained using intersecting stacks of D6-branes, there exist spectator sets of D6-branes whose presence is only required by tadpole cancellation, but which lead to additional gauge factors and matter content. Substitution of these D6-branes by a $\lambda H_{\rm NS}$ flux in the appropriate homology class would improve the models (see last reference in [19]).

A more interesting application of this idea would be to use the replacement of branes by fluxes to avoid certain chiral, but anomaly-free combinations of fields present in the model. For instance, the Standard Model like construction in [20] contains three standard quark-lepton generation plus an additional chiral but anomaly-free set of exotic fields. It would be interesting to get rid of these fields by replacing some of the D-branes by suitable fluxes.

4. Brane-flux transitions and the fate of chiral fermions

The topological couplings (2.8) show that certain combinations of NS-NS and RR fluxes are endowed with charge under RR fields, mimicking the coupling of a Dp-brane. This fact supports the possibility of transitions between branes and configurations of suitable fluxes, in a way consistent with charge conservation.

Such transitions indeed occur, and are typically non perturbative, i.e. the configurations have to tunnel through some potential barrier, see [26] for a nice description of one such effect. However, and even though it has not been exploited in the literature, we would like to emphasize that in some instances a continuous interpolation is also possible. To see that, consider the M-theory lift of a IIA process where an instanton dissolved into a stack of D6-branes turns into a dynamical D2-brane, via a continuous small instanton transition. In the M-theory lift, a multi-Taub-NUT space with a 4-form flux proportional to a harmonic form in Taub-NUT times an instanton emits the flux as a dynamical M2-brane, in a process we may refer to as "small fluxon transition". By performing a 9-11 flip in the above process, it can be described purely in terms of fluxes and branes in string theory.[3]

[3] One may worry that the instanton configuration involves gauge non-abelian degrees of freedom, hence involves non-perturbative states wrapped on collapsing cycles. However, one may discuss the transition involving only abelian instantons, obtained by compactifying additional dimensions in two two-tori and turning on (abelian) magnetic fields on them.

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A similar but non-supersymmetric transition takes place in the 9-11 flipped version of the process where a D4 brane along say 01234 approaches a D6-brane along 0123456 and is dissolved into a vortex-like magnetic flux in the D6-brane worldvolume. In the flipped version, a NS5-brane approaches a Taub-NUT space and is dissolved into a G_4 flux of the form $\omega \wedge F$, where ω is the harmonic 2-form in Taub-NUT and F denotes the gauge field in the vortex configuration.

Hence brane-flux transitions occur, either by tunneling or in a continuous fashion. Given this, it is natural to expect that in the presence of additional D-branes (probes) they may induce transitions where chiral fermions disappear from the probe world-volume, and explicit flux-induced Wess-Zumino terms arise, in order to maintain the world-volume anomaly cancellation. These processes interpolate between figures $1a$ and $1b$.

It is straightforward to provide explicit example of the above process, by considering transitions in F-theory compactifications on four-folds where some D3-branes are turned into fluxes. Introducing D7-brane probes as in section 3.1 would lead an explicit construction where chiral fermions disappear from the two-dimensional field theory, leaving behing a compensating flux-induced explicit WZ term. One may operate similarly in configurations of the kind studied in section 3.2, or other analogous construction.

It is interesting to compare the situation with other chirality changing phase transitions. Phase transitions in string theory in which the chiral content changes by (dis)appearance of fields in a non-anomalous representation have been studied in [27, 28, 20]. Since the anomaly structure of the theory is unchanged, such transitions do not require any compensating WZ terms. It is amusing that our flux-induced mechanism allows the occurrence of transitions in which the dynamical degrees of freedom which (dis)appear transform in anomalous representations.

5. Global gauge anomalies

It is easy to device configurations of branes and fluxes which lead to flux-induced global gauge anomalies on the D-brane world-volume. In this section we discuss one such example. The construction is closely related to the question of describing fluxes with RR charge classified by K-theory but not by cohomology.

The construction is based on the observation that global gauge anomalies due to dynamical fermions may arise at intersections of D-branes with D-branes carrying discrete K-theory charge. Consider a system introduced in [23]. We take type-I compactified on $\mathbb{T}^2$ with non-BPS D7-branes transverse to it. Each of the latter carries a $\mathbb{Z}_2$ charge in K-theory [29].[4] The discrete charge can be detected by introducing probes, given by a stack of D5-branes wrapped on $\mathbb{T}^2$. At the intersection of the D5-branes with each non-BPS D7-brane there arises a four-dimensional Weyl fermion in the fundamental of the USp group on the D5-branes, hence generating a global gauge anomaly. Consistency requires the number of D7-branes to be even, corresponding to the cancellation of RR charge in full K-theory [23].

[4]The instability of the D7-brane to decay to a bundle on the background D9-branes [30, 31] is irrelevant, since it will not be present in the T-dual model of interest studied below.

Let us consider a T-dual version of this construction, namely type IIA on a $\mathbb{T}^5$ (with 56789 compactified) with $O6^-$-planes along 013456 and D4 - anti-D4 pairs along 01234 (they may be separated in 789 from the O6-planes). This system again carries a non-trivial K-theory charge, which can be detected by introducing D8-brane probes along 012356789. The D8-brane gauge group is symplectic and each four-dimensional intersection with a D4 (and its image intersection with an anti-D4) leads to a four-dimensinoal Weyl fermion in the fundamental, hence to a $\mathbb{Z}_2$ global gauge anomaly.

Our last step is to transform some branes into fluxes using ideas in section 4. In fact, considering a D4 - anti-D4 brane well separated from the O6-plane, there is no topological obstruction to transforming the D4-brane into a $F_2\, H_{\rm NS}$ flux, and the anti-D4-brane into its image flux. They carry opposite charge under the RR 5-form C_5, as they should since it is odd under the orientifold action. However the construction guarantees that the combination of both fluxes carries a non-trivial $\mathbb{Z}_2$ K-theory charge. Using straightforward arguments about cancellation of K-theory charge in the compact space, or cancellation of global gauge anomalies on the D8-brane four-dimensional non-compact world-volume, one concludes there exists a flux-induced $\mathbb{Z}_2$ global gauge anomaly on the D8-brane worldvolume. If the number of pairs turned into fluxes is odd, the overall flux-induced global gauge anomaly is cancelled by the dynamical fermions arising from the remaining intersections.

Unfortunately the lack of a simple description of world-volume couplings to the discrete pieces of RR-fields when described in K-theory does not allow for a more explicit description of this effect. Hopefully the formal developments in [32] may help in addressing this issue. It would also be interesting to find a more general description (or other examples) of fluxes carrying discrete K-theory charge.

6. Origin of chirality in string theory

We would like to conclude with a different viewpoint on the relevance of the mechanism we have described in this paper. One of the basic features of Particle Physics is chirality, and hence one very interesting question in String Phenomenology is what String Theory mechanism reproduces four-dimensional chirality. There are several known mechanism to generate four-dimensional chiral fermions out of different string/M theory constructions, some of which have been exploited in building models with semi-realistic gauge and chiral matter spectrum. Among them, we would like to mention:[5]

a) Compactification of high dimensional gauge interactions with non-zero index of the Dirac operator (that is, with non-trivial holonomy in the compactification space, or non-trivial gauge bundle over it). The prototypical example is heterotic or Horava-Witten compactifications on Calabi-Yau threefolds [34].

b) Type-II/I D-branes wrapped on manifolds with non-trivial tangent/normal bundle and/or non-trivial gauge bundle. An example is provided by type-I compactifications on Calabi-Yau threefolds (e.g. [35]).

[5] v2. A further class of models with chiral matter can be obtained by F-theory compactifications with flux, see e.g. [33]. These are related to heterotic compactifications on Calabi-Yau threefolds via heterotic/F-theory duality.

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c) D-branes at singularities, the prototypical example being D3-branes at three-fold orbifold singularities in the transverse space [36].

d) D-branes wrapped on intersecting cycles, the chiral fermions arising at such intersections [15]. The prototypical case is compactifications with D6-branes wrapped on 3-cycles in the internal space.

e) Isolated G_2 holonomy singularities in M-theory, where chiral fermions arise from M2-branes wrapped on collapsed 2-cycles [37].

f) Configurations of NS5-branes and D5-branes, known as brane box and brane diamond models [38].

g) D4-branes in the presence of intersections of D6-branes and NS5-branes [39]

These different mechanisms give rise to chirality in D-brane world-volume in a similar way, namely via the appearance of chiral fermions in the spectrum. In fact it is easy to argue that, despite their superficial differences, these mechanism are closely related, in particular they are often related by string dualities,[6] hence should have a unified description in whatever microscopic theory underlies string theory.[7] This underlying equivalence, previously unnoticed in the literature, has a very satisfactory implication. Namely that the question of what is the "right" mechanism in string theory to reproduce the chirality of Particle Physics turns into the question of what of the above is the most efficient description of the unique underlying mechanism (say, for the value at which the moduli happen to stabilize). This is reminiscent of the argument by which duality turns the problem of selecting a string theory as the "right" one into a problem of moduli stabilization in a particular perturbative limit.

In this respect, the flux-induced anomaly mechanism we have described in the present paper provides a qualitatively different source of chirality in string theory. The fact that it is qualitatively different is clear in that it does not imply a dynamical fermion degree of freedom in the spectrum, hence no duality can relate it to the above mechanisms. In fact, it would be interesting to apply duality relations to our D-brane configurations to discover dual realizations of the mechanism. Interestingly, we have seen that even though there is no equivalence to the above mechanisms, the flux-induced anomaly is connected to them by brane/flux transitions, as discussed in section 4. This is reminiscent of the way different compactifications of a string theory are connected by topology-changing transitions. Hence, in a sense it turns the problem of what is the origin of chirality into a choice out of two topologically different mechanisms, which are neverthelss related by physical transition.

[6] I am grateful to Tomás Ortín for raising this question.

[7] In a sketchy way, models in a) and b) are often related by heterotic/type-I duality; models in c) are often mapped to b) by T-duality; models in d) are related to b) and c) by mirror symmetry; the M-theory lift of models in d) belong to class e); models in f), g) are T-dual to D-branes in certain orbifold and orientifold singularities [40] in c).

Acknowledgments

I thank L.E. Ibáñez for useful discussions and the ITP, Santa Barbara, for hospitality during completion of this work. I also thank M. González for kind support and encouragement. This work is supported by the Ministerio de Ciencia y Tecnología (Spain) through a Ramón y Cajal contract. This research was also supported in part by the National Science Foundation under Grant No. PHY99-07949.

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